Alg. 2 Warm Up # 2-5

Solve:

1) $x^2 - 5x - 14 > 0$

2) $5\left|2x - 1\right| \leq 30$

METHODS AND MEANINGS

p. 219

MATH NOTES

Notation for Inverses

When given a function $f(x)$, the notation for the inverse of the function is $f^{-1}(x)$. Note that the -1 is not a negative exponent. It is the mathematical symbol that indicates the "undo" or inverse function of $f(x)$.

For example, if $f(x) = x^3 - 1$ then $f^{-1}(x) = \sqrt[3]{x+1}$.

This same inverse notation is used to identify the inverse of trigonometric functions. For example the inverse of $\sin(x)$ is written $\sin^{-1}(x)$.

Graphs of inverses are a reflection in the line $y = x$

Inverse writing shortcut:

1) Swap x & y
2) Solve for y

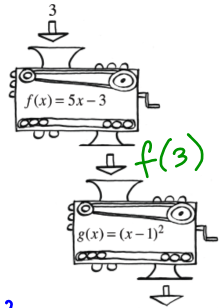
Review & Preview

HW Questions:

5-48. Two function machines, $f(x) = 5x - 3$ and $g(x) = (x-1)^2$, are shown at right.

a. Suppose $f(3)$, (*not* $x = 3$), is dropped into the $g(x)$ machine. This is written as $g(f(3))$. What is this output?

b. Using the same function machines, what is $f(g(3))$? Be careful! The result is different from the last one because the *order* in which you use the machines has been switched! With $f(g(3))$, first you find $g(3)$, then you substitute that answer into the machine named $f(x)$.



a) $g(f(3)) = (f(3) - 1)^2$

input

$((5 \cdot 3 - 3) - 1)^2$

$(12 - 1)^2$

$= 121$

b) $f(g(3))$

$5((3-1)^2) - 3$

17

5-49. This problem is a checkpoint for multiplying polynomials. It will be referred to as Checkpoint 5A.

Multiply and simplify each expression below.

a. $(x+1)(2x^2 - 3)$
b. $(x+1)(x^2 - 2x + 3)$
c. $2(x+3)^2$
d. $(x+1)(2x-3)^2$

$\frac{2x}{5} = \frac{138}{3}$

Now cross multiply

$\frac{4x-1}{x+1} = \frac{x-1}{1}$

cross multiply

$4x - 1 = x^2 - 1$

$0 = x^2 - 4x$

$= x(x-4)$

$\therefore$

5-50. Solve each of the following equations.

a. $\frac{3x}{5} = \frac{x-2}{4}$
b. $\frac{4x-1}{x} = 3x$
c. $\frac{2x}{5} - \frac{1}{3} = \frac{137}{3}$
d. $\frac{4x-1}{x+1} = x - 1$

$+\frac{1}{3}$ $+\frac{1}{3}$

5-51. Find the inverse of each of the following functions by first switching x and y and then solving for y.

a. $y = x^2 + 3$
b. $y = \left(\frac{1}{4}x + 6\right)^3$
c. $y = \sqrt{5x - 6}$

inverse: $x = \left(\frac{1}{4}y + 6\right)^3$

$\sqrt[3]{x} = \sqrt[3]{\left(\frac{1}{4}y + 6\right)^3}$

$\sqrt[3]{x} = \frac{1}{4}y + 6$

keep going

$y =$

5-52. Complete the square (for x) to write the equation that follows in graphing form and sketch the graph of $x^2 + y^2 - 4x - 16 = 0$. What is the parent graph and how has it been transformed?

$x^2 + y^2 = r^2$

5-53. Ever eat a maggot? Guess again! The FDA publishes a list, the Food Defect Action Levels list, which indicates limits for "natural or unavoidable" substances in processed food (*Time*, October 1990). So in 100 grams of mushrooms, for instance, the government allows 20 maggots! The average batch of rich and chunky spaghetti sauce has 350 grams of mushrooms. How many maggots does the government allow in a batch?

$\frac{20}{100} = \frac{m}{350}$

$(x-h)^2 + (y-k)^2 = r^2$

$\left(-\frac{4}{2}\right)^2$

$x^2 - 4x + \underline{4} + y^2 = 16 + \underline{4}$

$(x-2)^2 + (y-0)^2 = 20$

center: $(2, 0)$

$r = \sqrt{20}$

$r = \sqrt{4}\sqrt{5}$

$2\sqrt{5}$

5-54. Perform each operation below and simplify your results.

a. $\frac{x^2+4x+3}{x^2+3x} \cdot \frac{3x}{x+1}$

b. $\frac{y^2}{y+4} - \frac{16}{y+4}$

c. $\frac{x^2+x}{x^2-4x-5} \div \frac{3x^2}{x-5}$

d. $\frac{x^2-6x}{x^2-4x+4} + \frac{4x}{x^2-4x+4}$

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b. $\dfrac{y^2}{y+4}-\dfrac{16}{y+4}$

c. $\dfrac{x^2+x}{x^2-4x-5}\div\dfrac{3x^2}{x-5}$

d. $\dfrac{x^2-6x}{x^2-4x+4}+\dfrac{4x}{x^2-4x+4}$

$$\frac{x^2+x}{x^2-4x-5}\cdot\frac{x-5}{3x^2}$$

$$\frac{x^2-2x}{x^2-4x+4}$$

Yesterday's CP's: Continue working...

5-43. Statler, Adriena's teammate, is always looking for shortcuts. He thinks he has a way to adapt Adriena's strategy, but wants to check with his team before he tries it. *"If I use her strategy but instead of using a number, I skip a step and put the expression $f(x)$ directly into $g(x)$ to create $g(f(x))$, will I still be able to show that the equations are inverses?"*

a. What do you think about Statler's changes? What can you expect to get out?

b. Try Statler's idea on your team's equations, $f(x)$ and $g(x)$.

c. Describe your results.

d. Does Statler's strategy show that the two equations are inverses? How?

from #41

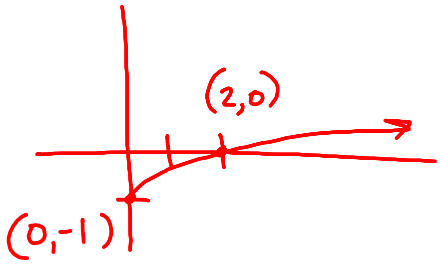
$f(x)=\sqrt{x-2}+3 \qquad g(x)=(x-3)^2+2$

$$g(f(x)) = (\sqrt{x-2}+3-3)^2+2$$
$$= (\sqrt{x-2})^2+2$$
$$= x-2+2$$
$$= x$$

$$f(g(x)) = \sqrt{(x-3)^2+2-2}+3$$
$$= \sqrt{(x-3)^2}+3$$
$$= x-3+3$$
$$= x$$

5-44. Trejo says that if you know the x-intercepts, y-intercepts, domain, and range of an equation then you automatically know the x-intercepts, y-intercepts, domain, and range for the inverse. Hilary disagrees. She says you know the intercepts but that is all you know for sure. Who is correct? Justify your answer.

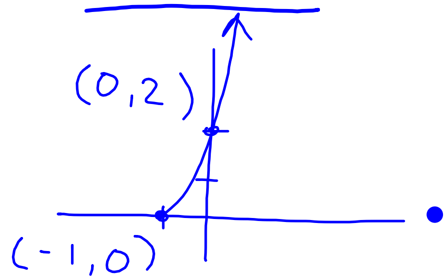
Example



dom: $x \geq 0$

range: $y \geq -1$

Inverse



dom: $x \geq -1$

range: $y \geq 0$

Classwork Week 2

Staple together with:

Warm Up on top

5- #1---> 5 (own paper)

5- #16 ---> 18
(with resource pages)

5- #20 ---> 23

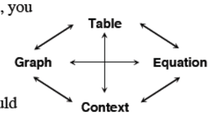
5- # 40 ---> 44

CP's: 5- #55 ---> 58 (pink)

5.2.1 How can I undo an exponential function?

Finding the Inverse of an Exponential Function

When you first began investigating exponential functions, you looked at how their different representations were interconnected, as in the web at right. So far in this chapter, you have considered how functions and their inverses are related in different representations including equations, $x \rightarrow y$ tables, and graphs. What would the inverse equation for each of the parent functions you worked with in Chapter 2 look like in each representation?



As you work with your team today, ask each other these questions:

What does the parent function look like in this representation?
How can that help us see the inverse relation?

Would another representation be more helpful?

How can we describe the relationship in words?

5-55. So far, you have worked with eight different parent graphs:

i. $y = x^2$ *ii.* $y = x^3$ *iii.* $y = x$ *iv.* $y = |x|$

v. $y = \sqrt{x}$ *vi.* $y = \frac{1}{x}$ *vii.* $y = b^x$ *viii.* $x^2 + y^2 = 1$

a. For each parent, find its inverse, if possible. If you can, write the equation of the inverse in $y =$ form. Include a sketch of each parent graph and its inverse. Remember that you can use the DrawInv function on your graphing calculator to help test your ideas.

b. Are any parent functions their own inverses? Explain how you know.

c. Do any parent functions have inverses that are not functions? If so, which ones?

5-56. THE INVERSE EXPONENTIAL FUNCTION

There are two parent functions, $y = |x|$ and $y = b^x$, that have inverses that you do not yet know how to write in $y =$ form. You will come back to $y = |x|$ later. Since exponential functions are so useful for modeling situations in the world, the inverse of an exponential function is also important. Use $y = 3^x$ as an example. Even though you may not know how to write the inverse of $y = 3^x$ in $y =$ form, you already know a lot about it.

a. You know how to make an $x \rightarrow y$ table for the inverse of $y = 3^x$. Make the table.

b. You also know what the graph of the inverse looks like. Sketch the graph.

c. You also have one way to write the equation based on your algebraic shortcut that you used in part (d) of problem 5-40. Write an equation for the inverse, even though it may not be in $y =$ form.

d. If the input for the inverse function is 81, what is the output? If you could write an equation for this function in $y =$ form, or as a function $g(x) =$, and you put in any number for x, how would you describe the outcome?

5-57. AN ANCIENT PUZZLE

Parts (a) through (f) below are similar to a puzzle that is more than 2100 years old. Mathematicians first created the puzzle in ancient India in the 2nd century BC. More recently, about 700 years ago, Muslim mathematicians created the first tables allowing them to find answers to this type of puzzle quickly. Tables similar to them appeared in school math books until recently.

Here are some clues to help you figure out how the puzzle works:

$\log_2 8 = 3$ $\log_3 27 = 3$

$\log_5 25 = 2$ $\log_{10} 10{,}000 = 4$

Use the clues to find the missing pieces of the puzzles below:

a. $\log_2 16 = ?$ b. $\log_2 32 = ?$ c. $\log_? 100 = 2$

d. $\log_5 ? = 3$ e. $\log_? 81 = 4$ f. $\log_{100} 10 = ?$

5-58. How is the Ancient Puzzle related to the problem of the inverse function for $y = 3^x$ in problem 5-56? Show how you can use the idea in the Ancient Puzzle to write an equation in $y =$ form or as $g(x) =$ for the inverse function in problem 5-56.

HW: 5-

60 ---> 67